(i) Let $u' = \alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_n u_n$

$u'' = \beta_1 u_1 + \beta_2 u_2 + \dots + \beta_n u_n$

$$T(\alpha u' + \beta u'') = T\left(\alpha \sum_{i=1}^{n} \alpha_i u_i + \beta \sum_{i=1}^{n} \beta_i u_i\right)$$

$$= T\left(\sum_{i=1}^{n} (\alpha\alpha_i + \beta\beta_i) u_i\right)$$

$$= \sum_{i=1}^{n} (\alpha\alpha_i + \beta\beta_i) v_i$$

$$= \sum_{i=1}^{n} \alpha\alpha_i v_i + \sum_{i=1}^{n} \beta\beta_i v_i$$

$$= \alpha \sum_{i=1}^{n} \alpha_i v_i + \beta \sum_{i=1}^{n} \beta_i v_i$$

$$= \alpha T(u') + \beta T(u'').$$

$\therefore$ T is linear.

* $u' + u'' = (\alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_n u_n) + (\beta_1 u_1 + \beta_2 u_2 + \dots + \beta_n u_n)$

$= (\alpha_1 + \beta_1) u_1 + (\alpha_2 + \beta_2) u_2 + \dots + (\alpha_n + \beta_n) u_n$

$T(u' + u'') = (\alpha_1 + \beta_1) v_1 + (\alpha_2 + \beta_2) v_2 + \dots + (\alpha_n + \beta_n) v_n$

$= (\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n) + (\beta_1 v_1 + \dots + \beta_n v_n)$

$= T(u') + T(u'')$

$\alpha u' = \alpha\alpha_1 u_1 + \alpha\alpha_2 u_2 + \dots + \alpha\alpha_n u_n$

$T(\alpha u') = \alpha\alpha_1 v_1 + \alpha\alpha_2 v_2 + \dots + \alpha\alpha_n v_n$

$= \alpha(\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n)$

$= \alpha T(u').$

$\therefore$ T is linear.